Extension of Cavallo et al.(2023) to heterogeneous exposures to energy price shocks

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1 Introduction & literature review

Understanding recent inflation spike episodes, especially following post-Covid energy price shocks, requires a better understanding of the dynamics of inflation and its propagation throughout the economy. Doing so necessitates the study of firms' price-setting process, adaptation to input prices' shocks and the frictions at play in such process, which are suspected to be the source of recent inflation. It is particularly important to understand how inflationnary pressures linked to the rise in some prices (that of energy here) can propagate in the economy and lead to overall price increases, and to which magnitude. Therefore, analysing the pass-through of such energy price shocks into the economy (and overall inflation) should help us understand the link between those two, and to what extent one was linked to the other.

A great range of models exist to take into account the frictions faced by firms when they try to adjust their prices (nominal rigidities). However they can be mostly organized in two separate categories, nesting from two different traditions : the time-dependent models on one hand, and the state-dependent models on the other. First, time-dependent models, such as the well-known Calvo, are based upon the idea that price adjustments possibilities arise only from time to time, and this frequency of "adjustment window" is what determines the frequency and size of price adjustments. In the famous Calvo model for instance, at each period there is a probability λ that a firm adjusts price, which is constant and does not depend on past shocks. These are usually very tractable models and are thus widely used in the macro literature. Furthermore, they fit the data quite well in low-inflation contexts. The issue arises when the economy is faced with large shocks and inflation spikes. In that case, these models do not take into account the size of the shock and thus the plausibility of an increase of price adjustments in the economy.

State-Dependent Models That is what the family of state-dependent models aims to address, by focusing, as their name indicates, on the state of a firm at each period of time. Generally in those models adjusting prices is costly, so the firms that adjust their price are those for whom the adjustment is most profitable. As a result, it appears that the probability of adjusting its price is completely dependent on the state of the firm (how far it is from its optimal markup). The underlying concept in a lot of these models is that of menu costs, precisely those costs that firms have to pay in order to be able to re-price.

As an illustration, Costain and Nakov (2011) introduce Generalized State-dependent Pricing (SDP), where the probability that a firm adjusts its prices is a non-decreasing function of the net value of adjusting (whether it is profitable or not). The two extreme cases are :

- If this probability is constant : The model is then akin to a Calvo model

- If the probability is either 0 (when value of adjusting is negative) or 1 (when the value of adjusting is positive) : It is a Golosov and Lucas' (2008) type menu cost model

Between those two extreme cases existing a wide range of state-dependent models, with different degrees of state-dependency.

The "pure menu cost" models of the type of Golosov & Lucas (2008), rely on the existence of a fix real cost of adjustment (menu cost k), that is often presented as a number of hours of labor needed to change price. Therefore, firms reprice if they are too far from their optimal price, and the inaction region is where, considering menu cost k, you are too close to the optimal price to want to reprice.

Nuances of SDP models Nevertheless, this only corresponds to a benchmark pure menu cost model, and a lot of more flexible SD models exist. One of the reason for relaxing some of Golosov & Lucas assumptions is that their model do not fit the small price adjustments that are observed in the data (Midrigan (2011)). Indeed, in the case of a fix menu cost, there is no reason why a firm would do very small price adjustments. That is why some decided to relax this assumption of purely fix menu cost, with the introduction of randomness in the size of the menu cost being iid from a period to another (Dotsey, King, Wolman (1999), Burstein (2006)). Nakamura and Steinsson (2010) also proposed a Calvo-Plus model, where menu costs can be either high or low, with some fix probability λ (hence the mention to Calvo). This adds randomness, and the possibility of small adjustments in the event of low menu costs. This model was, for instance, used by Gautier & Le Bihan (2022) in their analysis of price rigidity and monetary policy and a multisector model, with the specification of a low menu cost = 0.

Another way to relax some of the assumptions of strict menu cost models, is to look at a (series of) seminal paper by Caballero, Engel (1993), where the probability that a firm adjusts its price depends on the sign and the magnitude of the deviation of the price from its target level. They insist that there is no independence between the deviation from the target price and the probability of an adjustement (unlike in Calvo). Nevertheless, the way they get to it is not the same as in Golosov & Lucas who assume the highest degree of state-dependence. They introduce what they call an adjustment hazard function (Λ).

It is a flexible function describing the probability of individual price adjustment as a function of the magnitude of the deviation from optimal.

This is what was taken as a model, developed and refreshed by Cavallo et al. (2023), who developed a tractable version of this model. This is the model that will be used here as a baseline to build upon and analyse the price adjustment dynamics in a context of marginal cost (MC) increase through an energy prices' spike.

2 The model's economy

Let us first describe the model's framework and solve for the problems and constraints faced by the different actors and sectors of this economy, first under flex price, and then including rigidities.

2.1 Assumptions

2.1.1 Households

In this economy, households consume a composite good C made of a unit mass of varieties c_i with a constant elasticity of substitution $\eta > 1$

$$C_t = \left(\int (A_{it} c_{it})^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$$

The household problem is then to maximise utility :

$$\max_{C(t), c_i(t), H(t), M(t)} \int_0^\infty e^{-\rho t} \left(\frac{C_t^{1-\epsilon}}{1-\epsilon} - \alpha H_t + \log\left(\frac{M_t}{P_t} \right) \right) dt$$

where $\alpha > 0$ is a labor disutility parameter and $\rho > 0$ is the discount factor, subject to the inter-temporal budget constraint :

$$M_0 + \int_0^\infty Q_t \left(H_t W_t (1 + \tau_\ell) + \Pi_t + \tau_t - R_t M_t - \int_0^1 p_{it} c_{it} di \right) dt = 0$$

where R_t denotes the nominal interest rate, $Q_t = exp(-\int_0^t R_s ds)$ is the discount factor, W_t is the nominal wage rate, τ_ℓ a labor income tax, Π_t the firms' profits, τ_t a lump-sum transfer, p_{it} the nominal price of variety i and P is the price index.

2.1.2 Firms

This economy is populated by a unit mass of firms, indexed by $i \in [0, 1]$. A good c_i is produced by a firm i using labour h_i, firm-specific productivity Z_i and energy m_i as inputs, subject to the production technology $y_i = \left(\frac{h_i}{Z_i} \right)^{1-\xi_i} m_i^{\xi_i}$.

ξ_i is the energy share in production. Hence, firm i's marginal cost is $mc_{it} = K_i(W_t Z_t)^{(1-\xi_i)} E_t^{\xi_i}$, where E_t is the energy input price.

We assume that $Z_i(t) = exp(\sigma z_i(t))$, where {zi} are standard Brownian motions, independent across i, with standard deviation parameter σ.

2.2 Equilibrium conditions under flex price

By solving the FOC [1], we get the following nominal wage rate

$$W_t = e^{\mu_t} \frac{\alpha}{1+\tau_\ell} M_0(\rho + \mu)$$

It has a growth rate equal to μ which determines the trend inflation of the model (μ).

The demand for firm i's product is $c_i = \left(\frac{p_i}{P}\right)^{-\eta} A_i^{\eta-1} C$

Under flex-price, the optimal price, maximizing current profits, is $p^*_{it} = \frac{\eta}{\eta-1} mc_{it}$.

Then, the price index that would prevail under flex price is

$$P^*_t = \left(\int_0^1 \left(\frac{\eta}{\eta-1} \frac{mc_{it}}{A_{it}} \right)^{1-\eta} di \right)^{\frac{1}{1-\eta}}$$

and the optimal aggregate consumption is

$$C^*_t = \left(\frac{\alpha}{(1+\tau_\ell)\, W_t} P^*_t \right)^{-\frac{1}{\epsilon}}$$

To go further, as in CLM, let us introduce the firms' optimal (flex-price) market shares. We find that :

$$ms^*_{it} = \frac{\left(\frac{mc_{it}}{A_{it}}\right)^{1-\eta}}{\int_0^1 \left(\frac{mc_{it}}{A_{it}}\right)^{1-\eta}}$$

Finally, we can write firm's profits :

$$\Pi_i = (p_i - mc_i) \left(\frac{p_i}{P}\right)^{-\eta} A_i^{\eta-1} C$$

2.3 Introducing nominal rigidities

Let us now introduce nominal rigidities in the model. Due to these frictions, p_{it} is not necessarily equal to p^*_{it}, and such is the case for the price indices.

First, let us define the price gap $x_{it} \equiv \log\,(p_{it}/p^*_{it})$. This is the (log) deviation of a firm's price with respect to its optimal price p^*_t at each period t. Note there that the price of firm i can be expressed as $p_{it} = p^*_{it} e^{x_{it}}$.

[1] All proofs and detailed computations to be found in the Appendix

Price index Then, by solving the FOC, the price index is given by

$$P_t^{1-\eta} = \int_0^1 \left(\frac{\eta}{\eta - 1} \frac{mc_{it} e^{x_{it}}}{A_{it}} \right)^{1-\eta} di$$

The ratio of the actual price index over the optimal price index can thus be expressed as a function of the cross-section of the price gaps x and the optimal market shares ms_{it}^* :

$$\frac{P_t}{P_t^*} = \left(\int_0^1 e^{(1-\eta)x_{it}} ms_{it}^* di \right)^{\frac{1}{1-\eta}}$$

Consumption (aggregate) The aggregate consumption is :

$$C_t = \left(\frac{\alpha}{(1+\tau_\ell) W_t} \right)^{-\frac{1}{\epsilon}} (P_t^*)^{\frac{1}{\epsilon}} \left(\int_0^1 ms_{it}^* e^{(1-\eta)x_{it}} di \right)^{\frac{1}{\epsilon(\eta-1)}}$$

Which can also be written, with the price index :

$$C_t = \left(\frac{\alpha}{(1+\tau_\ell) W_t} P_t \right)^{-\frac{1}{\epsilon}}$$

And as a result, the ratio of aggregate consumption over its optimal level (prevailing in a flex price economy) is

$$\frac{C_t}{C_t^*} = \left(\frac{P_t}{P_t^*} \right)^{-\frac{1}{\epsilon}}$$

Firm's profit function : As we saw, $\Pi_{it} = (p_{it} - mc_{it}) \left(\frac{p_{it}}{P_t} \right)^{-\eta} A_i^{\eta-1} C_t$

$$\Pi_{it} = \left(\frac{p_{it}}{mc_{it}} - 1 \right) \left(\frac{p_{it}}{P_t A_i} \right)^{-\eta} A_i^{-1} C_t mc_{it}$$

We can now express profit, as a function of price gap $x \equiv \log{(p_i/p_i^*)}$, using the fact that $p_{it}^* = \frac{\eta}{\eta-1} mc_{it}$. Therefore, $\frac{p_{it}}{mc_{it}} = \frac{p_i t}{p_{it}^*} \frac{\eta}{\eta-1} = e^{x-it} \frac{\eta}{\eta-1}$ This gives :

$$\frac{\Pi(x_{it}, t)}{P_t} = \left[e^{x_{it}} - \frac{\eta - 1}{\eta} \right] e^{-\eta x - it} ms_{it}^* C_t^* \left(\int_0^1 e^{(1-\eta)x_{it}} ms_{it}^* di \right)^{\frac{1}{\epsilon(\eta-1)} - 1}$$

Cost function Let us define the flow cost function that represents forgone profits due to price gap x along a transition and in steady-state as

$$F(x,t,ms^*_{it}) \equiv 1 - \frac{\Pi(x,t,ms^*_{it})}{\Pi_{SS}(0)}$$

$\int_0^1 e^{(1-\eta)x} ms^*_{it} di = 1$ in steady-state, as long as the market share of the firm is orthogonal to the price gap x (if we consider that at steady-state all x are not 0). Simplifying and assuming C^*_t, and ms^*_{it} very close to their steady state value we can get :

$$F(x,ms^*_{it}) = 1 - \eta\left[e^x - \frac{\eta-1}{\eta}\right] e^{-\eta x}\left(\int_0^1 e^{(1-\eta)x} ms^*_{it} di\right)^{\frac{1}{\epsilon(\eta-1)}-1}$$

2.4 The firm's price-setting problem.

Let us now consider the firm's price-setting problem. In this case, we consider a permanent shock on energy prices, and thus a permanent MC shock for firms. This adds to the trend shift of the price gaps x_{it}.

Trend shift of x_t Following Cavallo et al., we assume that E_t follows (on average) inflation and therefore grows at the same rate as W_t, and that Z_{it} have a constant mean and therefore are expected to grow at constant rate 0. The trend shift of x_t is then :

$$x_t = log(p_t) - log(p^*_t)$$

$$\Delta x = x_{t+1} - x_t = log(p^*_t) - log(p^*_{t+1})$$

in the absence of adjustment

$$\Delta x = log(mc_{it}) - log(mc_{i,t+1})$$

$$\Delta x = log\left(K_i Z_{it} W_t \left(\frac{E_t}{Z_{it}W_t}\right)^{\xi_i}\right) - log\left(K_i Z_{i,t+1} W_{t+1}\left(\frac{E_{t+1}}{Z_{i,t+1}W_{t+1}}\right)^{\xi_i}\right)$$

Assuming E_t follows (on average) inflation and therefore grows at the same rate as W_t, and that Z_{it} have a constant mean and therefore are expected to grow at constant rate 0, the term $\left(\frac{E_{t+1}}{Z_{i,t+1}W_{t+1}}\right)$ is then constant. Therefore, the drift in x is just driven by the growth rate of W and Z :

$$\Delta x = log\left(\frac{Z_{it}}{Z_{i,t+1}}\right) + log\left(\frac{W_t}{W_{t+1}}\right)$$

$$dx_t = -\mu dt + \sigma dz_i(t)$$

where z_i is a standard Brownian motion describing productivity shocks and μ is the growth rate of nominal wages per unit of time, i.e. the inflation rate.

Price-setting problem The firm's sequential problem consists in minimizing the flow costs from forgone profits and effort costs by choosing hazard rates ℓ_t and the optimal reset point x^* according to

$$v(x) = \min_{\ell_t, x^*} E\left\{\int_0^\infty e^{-\rho t}\left[F(x_t) + (\kappa\ell_t^\gamma\right] dt \mid x(0) = x\right\}$$

$$\text{s.t. } x_t = x_0 - \mu t + \sigma z_t + \sum_{\tau_i < t} \Delta x(\tau_i)$$

where τ_i denotes the stopping times when a resetting opportunity arrives, $\Delta x(\tau_i) = x^* - x_{\tau_i}$ is the price change conditional on an adjustment. Indeed, each time a firm adjusts its price it goes back to the optimal price gap x^*, therefore the size of the adjustment is $\Delta x = x^* - x$. Therefore, x_t depends on the initial x_0, its 'trend shift' $(-\mu t + \sigma z_t)$ and the additional price (gap) changes that are done, at each stopping time τ_i of adjustment. Finally, $(\kappa \ell_t)^\gamma$ is the effort cost of choosing hazard rate ℓ, with $\kappa > 0$ and $\gamma > 1$

Note that for a given firm the permanent shock on energy prices will only affect x_0, shifting it and therefore affecting the initial condition of the firm's problem, without modifying the trend shift induced by inflation and productivity shocks. This will be treated in the next section.

3 Solution to the firm's price-setting problem and dynamics of the model

As shown by Cavallo et al., at a second-order Taylor approximation around the steady state implied by the sequential formuation of the price setting problem, the firm's value function $v(x)$ solves the following problem :

$$\rho v(x) = F(x) - \mu v'(x) + \frac{\sigma^2}{2} v''(x) + \min_{\ell \geq 0}\left\{\ell \cdot \left(v\left(x^*\right) - v(x)\right) + (\kappa\ell)^\gamma\right\}$$

where x^* is the optimal price gap that is chosen in case of adjustment, satisfying $v'(x^*) = 0$.

Then, minimizing the right hand-side optimization program, we get an optimal adjustment effort ℓ^* for each x, satisfying :

$$\left(v\left(x^*\right) - v(x)\right) + \kappa\gamma\left(\kappa\ell^*\right)^{\gamma-1} = 0$$

$$(\kappa\ell^*)^{\gamma-1} = \frac{v(x) - v\left(x^*\right)}{\kappa\gamma}$$

$$\ell^* = \frac{1}{\kappa}\left(\frac{v(x) - v\left(x^*\right)}{\kappa\gamma}\right)^{\frac{1}{\gamma-1}}$$

Hence, we can write $\Lambda(x) = \ell^*$ the price adjustment probability (per unit of time) at any given x. This will be useful as it allows to study the dynamics of the model. Indeed, having the distribution of price gaps x, that we call $m(x,t)$ and the distribution of adjustment probability $\Lambda(x)$ we can compute the key statistics we are interested in, such as the frequency of price adjustments, their size, their dynamics after a shock, etc.

Post-shock distribution of x : At impact, a permanent MC shock shifts all the price gaps immediately. It does so in uneven ways however, as more exposed firms face larger shock. Contrarily to the original Cavallo et al. which only considers an aggregate shock, here the phenomenon can be split in two : the aggregate shock δ and the firm-specific exposure ξ_i. In this case, we consider that the exposure ξ_i and the initial price gap x_i are uncorrelated for all firms.

To make the study easier and more tractable, let us consider five quintiles of exposure ξ. The shifts are respectively $\delta\xi_q$, where ξ_q is the quintile-specific degree of exposure. Thus we get one distribution for each quintile:

$$\hat{m}_q(x,0) = m(x + \delta\xi_q)$$

meaning that, at impact, the shock shifts the initial distribution $m(x)$ to a new distribution $\hat{m}_q(x,0)$, a shift whose scale depends on ξ_q.

More generally, $\hat{m}(x,0)$ is the post-shock distribution of individual x_i having all faced an idiosyncratic shock of size $\delta\xi_i$. As the size of the idiosyncratic shock fully depends on the firm's exposure ξ_i, it may well be that two firms with the same initial price gap x shift to two different points in the post-shock distribution $\hat{m}(x,0)$.

4 Computing the pass-through

After a shock, firms will adjust their prices optimally, setting them such that $x = x^*$. As a result, at the firm level, every adjustment is *optimal* and corresponds to a 'full pass-through'. Therefore, to find the overall pass-through, it should be sufficient to look at the share of adjusting firms, at each period.

In the original Cavallo et al. we have $\hat{m}(x,t)$ the distribution of price gaps x after a shock, $\Lambda(x)$ is the adjustment probability at any given x. To compute the period pass-through let us find the share of adjusting firms at t:

$$pt(t+dt) = \int \hat{m}(x,t)\Lambda(x)\mathrm{dt}\ \mathrm{dx}$$

$\hat{m}(x,t)$ is adjusting at every period t. $\Lambda(x)dt$ is therefore the adjustment rate at a given period.

If we were to compute the total or cumulative pass-through, through the cumulated adjustment rate, we would have :

$$CPT(t) = \int_0^t \int \hat{m}(x,s)\Lambda(x)\mathrm{dt}\ \mathrm{dx}\ ,$$

which is just the integration of all period adjustments from the shock (impact at $s = 0$) until period t.

This methodology raises several issues as, due to the existence of steady-state positive adjustment rates, there is an upward bias in the estimation of the pass-through, which can hardly be controlled for and adressed.

This is why, simulations are also done at the firm-by-firm level, looking at the precise date of adjustment of each of them.

4.1 Implications for the pass-through along the distribution of energy exposures and firm level analysis

Let us introduce briefly the methodology employed here, which differs from original: the idiosyncratic impact of the shock and the firm's response are simulated and observed for a large number of firms replicating the model's distribution described before. This allows to observe the exact adjustment date s of each firm, before to aggregate it over firms at an overall - or quintile - level. Once this aggregate is obtained it gives us the pass-through, and its dynamics across time. This can be done under different specifications, for instance looking at the aggregate level as well as at the level of quintiles of energy exposure.

In these simulations, we first use homogeneous and then heterogeneous A_i which lead to different levels of market share. In the heterogeneous case, they are drawn, respectively :

- from a uniform distribution, uncorrelated with the x
- from two uniform distributions (one with a mean much larger than the other), correlated with the x

We also draw heterogeneous levels of exposure ξ_i either by quintile (5 of them) or then from a continuous distribution (log-normal, following the data).

5 Results and interpretation of inter-quintile exposure difference

It is possible to study the difference in pass-through by quintile of exposure. To do so, one just has to order and group the firms by quintile of exposure and aggregate their individual adjustment dates to compute each quintile's cumulative pass-through. The main results are presented in the figures below.

First, let us look at the period-by-period, and cumulative, rate of adjustment (yielding the pass-through here). Following the method presented above, we can compute the period-by-period adjustment rates for each quintile of interest. Unsurprisingly, it appears that the firms with the higher exposure are the ones that immediately react the most to the shock, and their pass-through is faster. It also appears that each of them reaches a full pas-through, sooner or later. This result qualitatively depends on the parameterization that is used in the simulations, especially regarding the celerity of the pass-through.

To better understand the role of different parameters, and potentially prepare a future calibration and estimation of the model, it may be useful to look at the output of simulations based on different parameterization (of γ and κ) in a comparative statics exercise.

5.1 Comparative statics

In figure 1, 2 and 3 we can see the output of simulations done for several pairs of parameters. As Cavallo et al. had already remarked, γ is a shape parameter that eventually determines the state-dependence of the model under study. The higher the γ, the less state-dependent is the model, and as it tends to infinity, the model tend to a Calvo one. On the other hand, as $\gamma \to 1$, the model tends to a pure menu cost (Golosov & Lucas, 2008). This comes from the form of the adjustment hazard function that has been described in section 3. That is why the celerity of the pass-through, but also its shape and its heterogeneity across different levels of exposure, and different sizes of shock, are key moments that should allow to calibrate and estimate the model.

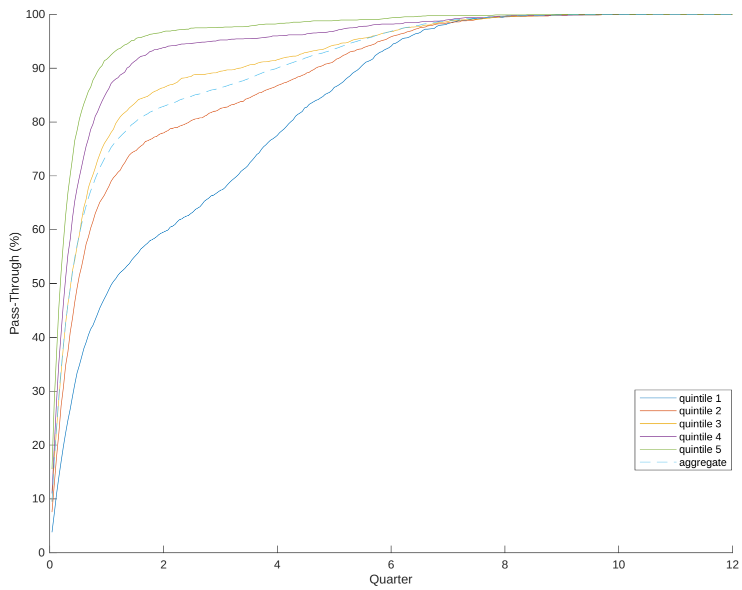

Figure 1: Pass-Through after the shock, for $\gamma = 2.22$ and $\kappa = 0.11$

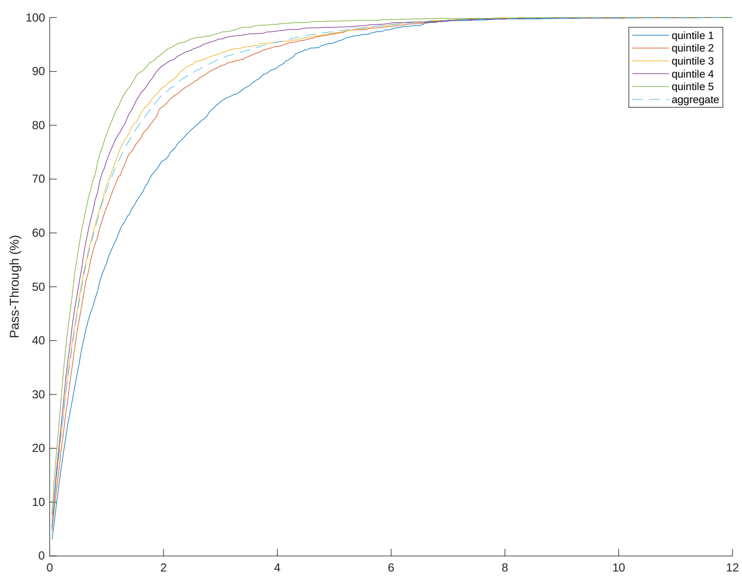

Figure 2: Pass-Through after the shock, for $\gamma = 5$ and $\kappa = 0.15$

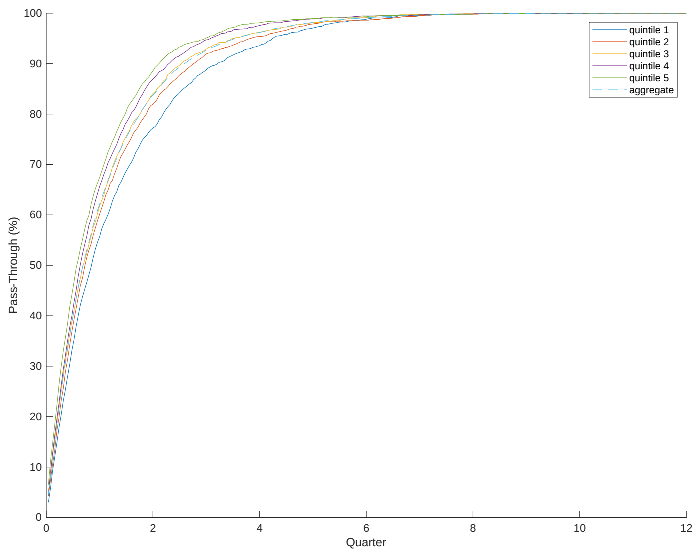

Figure 3: Pass-Through after the shock, for $\gamma = 10$ and $\kappa = 0.2$

References

Burstein, Ariel T. (2006). "Inflation and output dynamics with state-dependent pricing decisions." *Journal of Monetary Economics* 53.7, pp. 1235–1257.

Caballero, R. J. and E. M. Engel (1993). "Microeconomic rigidities and aggregate price dynamics" *European Economic Review* 37.4, pp. 697–711.

Cavallo, Alberto, Francesco Lippi, and Ken Miyahara (2023). *Large Shocks Travel Fast.* NBER Working Papers 31659. National Bureau of Economic Research, Inc.

Costain, James and Anton Nakov (2011). "Price Adjustments in a General Model of State-Dependent Pricing" *Journal of Money, Credit and Banking* 43.2-3, pp. 385–406.

Dotsey, Michael, Robert G. King, and Alexander L. Wolman (1999). "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output." *The Quarterly Journal of Economics* 114.2, pp. 655–690.

Gautier, Erwan and Hervé Le Bihan (July 2022). "Shocks versus Menu Costs: Patterns of Price Rigidity in an Estimated Multisector Menu-Cost Model" *The Review of Economics and Statistics* 104.4, pp. 668–685.

Golosov, Mikhail and Robert E. Lucas Jr. (2007). "Menu Costs and Phillips Curves" *Journal of Political Economy* 115.2, pp. 171–199.

Midrigan, Virgiliu (2011). "Menu costs, multi-product firms, and aggregate fluctuations." *Econometrica* 79.4, pp. 1139–1180.

Nakamura, Emi and Jón Steinsson (Aug. 2010). "Monetary Non-neutrality in a Multisector Menu Cost Model." *The Quarterly Journal of Economics* 125.3, pp. 961–1013.

6 Appendix

6.1 First order conditions

(1)

$$C_t : e^{-\rho_t} C_t^{-\epsilon} - \lambda Q_t P_t = 0$$

(2)

$$c_{it} : e^{-\rho_t} \left(A_{it}^{\frac{\eta}{\eta-1}} c_{it}^{-\frac{1}{\eta}} \right) C_t^{-\epsilon} C_t^{\frac{1}{\eta}} - \lambda Q_t pit = 0$$

(3)

$$H_t : -e^{-\rho_t} \alpha + \lambda Q_t W_t (1 + \tau_\ell) = 0$$

(4)

$$M_t : e^{-\rho_t} \frac{1}{P_t} \frac{P_t}{M_t} - \lambda Q_t R_t = e^{-\rho_t} \frac{1}{M_t} - \lambda Q_t R_t = 0$$

Combining (2)/(1) :

$$\frac{p_{it}}{P_t} = C_t^{\frac{1}{\eta}} A_{it}^{\frac{\eta-1}{\eta}} c_{it}^{-\frac{1}{\eta}}$$

$$\leftrightarrow c_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\eta} A_{it}^{\eta-1} C_t$$

6.2 Money supply and demand

Now, re-arranging (4) and differentiating with respect to time :

$$e^{-\rho_t} \left(-\frac{\dot{M}_t}{M_t^2} \right) - \rho e^{-\rho_t} \frac{1}{M_t} = \lambda (Q_t \dot{R}_t + \dot{Q}_t R_t)$$

$$-e^{-\rho_t} \frac{\dot{M}_t}{M_t^2} - \rho e^{-\rho_t} \frac{1}{M_t} = \lambda (Q_t \dot{R}_t - Q_t R_t^2)$$

where $\dot{Q}_t = -Q_t R_t$

Then, assume a monetary policy $M_t = M_0 \exp(\mu t)$ (as in CLM) and simplify :

$$-e^{-\rho_t} \frac{\mu M_0 \exp(\mu t)}{M_0^2 \exp(2\mu t)} - \rho e^{-\rho_t} \frac{1}{M_0 \exp(\mu t)} = \lambda Q_t \left(\dot{R}_t - R_t^2 \right)$$

$$-(\mu + \rho) \left(e^{-\rho_t} \frac{1}{M_0 exp(\mu t)} \right) = \lambda Q_t R_t \left(\frac{\dot{R}_t}{R_t} - R_t \right)$$

and by using (4) we are left with:

$$\mu + \rho = R_t - \frac{\dot{R}_t}{R_t}$$

which is solved by $R_t = \rho + \mu$, all t.
Then, replacing R_t and $Q_t = exp\,(-(\rho+\mu)t)$ in (4), we have : $\lambda = \frac{e^{-\rho_t}}{M_0 exp(\mu t) Q_t(\rho+\mu)}$
$\lambda = \frac{1}{M_0(\rho+\mu)}$

From equation (3), we can replace λ and the nominal wage rate is given by:

$$-e^{-\rho_t}\alpha + \frac{1}{M_0(\rho+\mu)} Q_t W_t (1+\tau_\ell) = 0$$

$$W_t Q_t (1+\tau_\ell) = e^{-\rho_t}\alpha M_0(\rho+\mu)$$

$$W_t = e^{\mu_t}\frac{\alpha}{1+\tau_\ell} M_0(\rho+\mu)$$

It has a a growth rate equal to μ (which is useful later, as it allows to explain the trend inflation in the model).
Then, using $c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\eta} A_{it}^{\eta-1} C_t$, and aggregating over varieties we get :

$$C_t = \left(\int_0^1 \left(A_{it}\left(\frac{p_{it}}{P_t}\right)^{-\eta} A_{it}^{\eta-1} C_t\right)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}$$

$$P_t^{-\eta} = \left(\int_0^1 \left(A_{it}^{\eta-1} p_{it}^{1-\eta} di\right)\right)^{\frac{\eta}{\eta-1}}$$

$$P_t^{1-\eta} = \int_0^1 \left(\frac{p_{it}}{A_{it}}\right)^{1-\eta} di$$

Then, let us use $x = log(p_t) - log(p_t^*)$, which can be re-arranged into
$p_t = p_t^* e^x = \frac{\eta}{\eta-1} mc_{it} e^x$
Then,

$$P_t^{1-\eta} = \int_0^1 \left(\frac{\eta}{\eta-1}\frac{mc_{it}e^x}{A_{it}}\right)^{1-\eta} di \tag{5}$$

At this point, let us also introduce P_t^* the price index that would prevail under flex price.

$$P_t^* = \left(\int_0^1 \left(\frac{\eta}{\eta-1}\frac{mc_{it}}{A_{it}}\right)^{1-\eta} di\right)^{\frac{1}{1-\eta}}$$

To go further, as in CLM, let us study and introduce the firms' optimal (flex-price) market shares into the price index. **This is done in details at the beginning of the firms' section**

We find that $ms^*_{it} = \frac{\left(\frac{\eta}{\eta-1}\frac{mc_{it}}{A_{it}}\right)^{1-\eta}}{\int_0^1\left(\frac{\eta}{\eta-1}\frac{mc_{it}}{A_{it}}\right)^{1-\eta}}$

Therefore, we can re-arrange (5), and then replace using ms^*_{it}:

$$P_t^{1-\eta} = \int_0^1 \left(\frac{\eta}{\eta-1}\frac{mc_{it}}{A_{it}}\right)^{1-\eta} e^{(1-\eta)x} di$$

$$P_t = \left(S_t \int_0^1 e^{(1-\eta)x} ms^*_{it} di\right)^{\frac{1}{1-\eta}}$$

$$\frac{P_t}{P_t^*} = \left(\int_0^1 e^{(1-\eta)x} ms^*_{it} di\right)^{\frac{1}{1-\eta}}$$

6.3 Consumption (aggregate)

Now, combining (2)/(3), we get:

$$(A_{it}c_{it})^{-\frac{1}{\eta}} A_{it} = \frac{\alpha p_{it}}{W_t(1+\tau_\ell)} C_t^{\epsilon-\frac{1}{\eta}}$$

$$A_{it}c_{it} = \left(\frac{p_{it}}{W_t A_{it}}\right)^{-\eta} \left(\frac{\alpha}{1+\tau_\ell}\right)^{-\eta} C_t^{1-\epsilon\eta}$$

where we can now use the same trick as for the price index, $p_{it} = p^*_{it}e^x = \frac{\eta}{\eta-1}mc_{it}e^x$ and be left with :

$$A_{it}c_{it} = \left(\frac{\eta}{\eta-1}\frac{mc_{it}e^x}{W_t A_{it}}\right)^{-\eta} \left(\frac{\alpha}{1+\tau_\ell}\right)^{-\eta} C_t^{1-\epsilon\eta}$$

Integrating over varieties, we get :

$$C_t^{\frac{\eta-1}{\eta}} = \int_0^1 (A_{it}c_{it})^{\frac{\eta-1}{\eta}} di = \int_0^1 \left[\left(\frac{\eta}{\eta-1}\frac{mc_{it}e^x}{W_t A_{it}}\right)^{-\eta} \left(\frac{\alpha}{1+\tau_\ell}\right)^{-\eta} C_t^{1-\epsilon\eta}\right]^{\frac{\eta-1}{\eta}} di$$

$$C_t^{\epsilon(\eta-1)} = \int_0^1 \left(\frac{\eta}{\eta-1}\frac{mc_{it}e^x}{W_t A_{it}}\right)^{1-\eta} \left(\frac{\alpha}{1+\tau_\ell}\right)^{1-\eta} di$$

Now we can use $ms^*_{it} = \frac{\left(\frac{\eta}{\eta-1}\frac{mc_{it}}{A_{it}}\right)^{1-\eta}}{\int_0^1\left(\frac{\eta}{\eta-1}\frac{mc_{it}}{A_{it}}\right)^{1-\eta}}$ and replace in the expression for C_t :

$$C_t^{\epsilon(\eta-1)} = \left(\frac{\alpha}{(1+\tau_\ell)\,W_t}\right)^{1-\eta} \int_0^1 (ms_{it}^* S_t)\, e^{(1-\eta)x} di$$

$$C_t = \left(\frac{\alpha}{(1+\tau_\ell)\,W_t}\right)^{-\frac{1}{\epsilon}} \left(S_t \int_0^1 ms_{it}^* e^{(1-\eta)x} di\right)^{\frac{1}{\epsilon(\eta-1)}}$$

Which can also be written with the price index :

$$C_t = \left(\frac{\alpha}{(1+\tau_\ell)\,W_t} P_t\right)^{-\frac{1}{\epsilon}}$$

And in the same way, $C_t^* = \left(\frac{\alpha}{(1+\tau_\ell)W_t} P_t^*\right)^{-\frac{1}{\epsilon}}$

7 Firms

7.1 Firm's market share

Let us derive a firm's market share $ms_i = \frac{c_{it}p_{it}}{C_t P_t}$:
By definition, $C_t = \left(\int_0^1 (A_{it}c_{it})^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}$. Then, we now know that $c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\eta} A_{it}^{\eta-1} C_t$ and $P_t = \left(\int_0^1 \left(\frac{p_{it}}{A_{it}}\right)^{(1-\eta)}\right)^{\frac{1}{1-\eta}}$, therefore :

$$ms_i = \frac{c_{it}p_{it}}{\left(\int_0^1 (A_{it}\left(\frac{p_{it}}{P_t}\right)^{-\eta} A_{it}^{\eta-1} C_t)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}} P_t}$$

$$ms_i = \frac{c_{it}p_{it}}{C_t P_t^\eta \left(\int_0^1 (A_{it}^{\eta-1} p_{it}^{1-\eta} di\right)^{\frac{\eta}{\eta-1}} P_t}$$

$$ms_i = \frac{\left(\frac{p_{it}}{P_t}\right)^{-\eta} A_{it}^{\eta-1} C_t p_{it}}{C_t P_t^\eta P_t^{-\eta} P_t}$$

$$ms_i = \frac{\left(p_{it}A_{it}^{-1}\right)^{1-\eta}}{\int_0^1 \left(p_{it}A_{it}^{-1}\right)^{1-\eta} di}$$

Let us now solve for the optimal market share, under flex price, where the optimal price $p^* = \frac{\eta}{\eta-1} mc$:

$$ms_i^* = \frac{\left(\frac{\eta}{\eta-1} mc_i A_i^{-1}\right)^{1-\eta}}{\int_0^1 \left(\frac{\eta}{\eta-1} mc_i A_i^{-1}\right)^{1-\eta}}$$

Let $S_t \equiv \int_0^1 \left(\frac{\eta}{\eta-1} mc_i A_i^{-1}\right)^{1-\eta} di$
Finally, introducing t in the market share

$$ms_{it}^* = \frac{\left(\frac{\eta}{\eta-1}\frac{mc_i}{A_i}\right)^{1-\eta}}{S_t}$$

Notice that $S_t = (P_t^*)^{1-\eta}$. Furthermore, the market share can now be re-written as a function of the price gap x, the optimal market share ms_{it}^* and the price indices:

$$ms_i = \frac{\left(p_{it}A_{it}^{-1}\right)^{1-\eta}}{P_t^{1-\eta}} = \left(\frac{\left(p_{it}A_{it}^{-1}\right)}{P_t^*}\frac{P_t^*}{P_t}\right)^{1-\eta}$$

$$ms_i = \frac{\left(\frac{\eta}{\eta-1} mc_i A_i^{-1} e^x\right)^{1-\eta}}{S_t}\left(\frac{P_t^*}{P_t}\right)^{1-\eta}$$

$$ms_i = ms_i^* e^{(1-\eta)x}\left(\frac{P_t^*}{P_t}\right)^{1-\eta}$$

Finally we can introduce $y \equiv log(ms) - log(ms^*)$, the (log-)deviation of the market share from optimal

$$y = log(ms^*) + log(e^{(1-\eta)x}) + (1-\eta)log\left(\frac{P_t}{P_t^*}\right) - log(ms^*)$$

$$y = (1-\eta)x + (1-\eta)log\left(\frac{P_t}{P_t^*}\right)$$

7.2 Firm's profit function

We can now express profit, as a function of price gap $x \equiv \log\,(p_i/p_i^*)$, and using the fact that $p_{it}^* = \frac{\eta}{\eta-1}mc_{it}$. Therefore, $\frac{p_i}{mc_i} = \frac{p_i}{p_i^*}\frac{\eta}{\eta-1} = e^x\frac{\eta}{\eta-1}$ This gives us:

$$\Pi(x,t) = \left(\frac{p_{it}}{mc_{it}} - 1\right)\left(\frac{\eta}{\eta-1}e^x\right)^{-\eta}\left(\frac{mc_{it}}{A_{it}}\right)^{1-\eta} P_t^\eta C_t$$

$$\Pi(x,t) = \left[e^x\frac{\eta}{\eta-1} - 1\right] e^{-\eta x} ms_{it}^* S_t \frac{\eta-1}{\eta} P_t^\eta C_t$$

$$\frac{\Pi(x,t)}{P_t} = \left[e^x - \frac{\eta-1}{\eta}\right] e^{-\eta x} ms_{it}^* \left(\frac{P_t}{P_t^*}\right)^{\eta-1} C_t^* \left(\frac{C_t}{C_t^*}\right)$$

Let us rewrite and simplify $\left(\frac{P_t}{P_t^*}\right)^{\eta-1}\frac{C_t}{C_t^*}$

$$= \left(\frac{P_t}{P_t^*}\right)^{\eta-1} \left(\frac{\alpha P_t}{(1+\tau_\ell)\, W_t} \frac{(1+\tau_\ell)\, W_t}{\alpha P_t^*}\right)^{-\frac{1}{\epsilon}}$$

$$= \left(\frac{P_t}{P_t^*}\right)^{\eta-1-\frac{1}{\epsilon}}$$

$$= \left(\int_0^1 e^{(1-\eta)x} ms_{it}^* di\right)^{\left(\frac{1}{1-\eta}\right)\left(\frac{\epsilon(\eta-1)-1}{\epsilon}\right)}$$

$$= \left(\int_0^1 e^{(1-\eta)x} ms_{it}^* di\right)^{\frac{1}{\epsilon(\eta-1)}-1}$$

We are thus left with :

$$\frac{\Pi(x,t)}{P_t} = \left[e^x - \frac{\eta-1}{\eta}\right] e^{-\eta x} ms_{it}^* C_t^* \left(\int_0^1 e^{(1-\eta)x} ms_{it}^* di\right)^{\frac{1}{\epsilon(\eta-1)}-1}$$

Remark *In CLM, they have the same equation for real profit. The difference is that (1) they "note" that all time dependent terms (P_t, C_t, mc_t) can be derived from the distribution of x while it is not exactly the case here, since all these terms depend also on the (optimal) market share, which is firm-specific and not fully summarized by the distribution of x. We could write $\Pi(x, t, ms_i)$, to make clear that Profit is a function of the price gap, time and the market share. (2) They notice that "the assumption $A_i = Z_i^{1-\xi_i}$ makes the profit function independent of the productivity shock, a feature that allows us to reduce the state space of the problem to a single scalar variable x."*
This simplification has not been done there, hence the apparition of market shares in the expression for real profit.

7.3 Steady-state adjustments and upward bias of the pass-through after a shock

In steady state, let us remind that x shifts as follows : $dx(t) = -\mu dt + \sigma dz_i(t)$
Therefore, at each period, due to trend inflation and to idiosyncratic productivity shocks, some firms are drifting far away from their optimal price gap x^* and therefore set a high repricing effort. As a result, at each period, there are some price adjustments. This is the case even in a zero-inflation steady-state, where the overall mean adjustment is null (because on average firms face a mean productivity shock equal to 0), but the frequency of adjustment is positive due to the dispersion of idiosyncratic productivity shocks.
This raises an issue in the estimation of a post-shock pass-through. Indeed, after a shock, an increasing number of firms adjust their prices. If we were to follow the method proposed above, we would take the share of adjusting firms

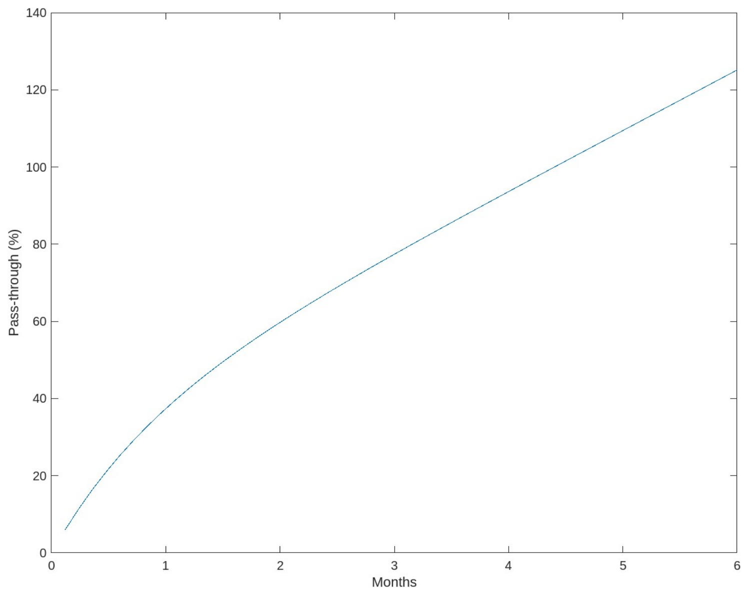

Figure 4: Cumulated adjustment rate after a shock

as the overall pass-through. Nonetheless, as time goes by, even after all firms have adjusted their prices, we would get back to the steady-state situation, where fewer but still some firms adjust at each period. Therefore, the adjustment rate would keep increasing and exceed 100 % (see Figure 1). Even though these adjustments would not correspond to firms adjusting their prices to "pass" their MC increase, and therefore not corresponding to our definition of the pass-through of firms' MC increase into their prices, they would be accounted for in the same way, conducting to an upward bias of the pass-through of the shock.

7.4 Attributing the additional pass-through to idiosyncratic shock

To remedy this, we can try to isolate the part of the adjustment which is attributable to the impact of the shock. This is the additional adjustment, compared to steady state, at each period. Let us write it properly :

$$\Delta pt(t) = \int \hat{m}(x,t)\Lambda(x)\Delta \text{s dx} \ - \int m_{ss}(x)\Lambda_{ss}(x)\Delta \text{s dx}$$

$$\Delta pt(t) = \int \hat{m}(x,t)\Lambda(x)\Delta \text{s dx} \ - \text{freq}_{ss}\Delta \text{s}$$

where freq_{ss} is the (yearly) frequency of price adjustment, at steady state.

Note that if we were to compute the cumulative pass-through instead of the period-specific adjustment, we would also need to integrate over distributions $\hat{m}(x, s)$ over time, for periods 0 to t.
Since $\Lambda(x)$ is (approximately) constant over time, this can also be written as

$$\Delta pt(t) = \int (\hat{m}(x,t) - m_{ss}(x))\, \Lambda(x) \Delta \mathrm{s}\ \mathrm{dx}$$

It is apparent here that the shift in the distribution of x at period 0 ($\hat{m}(x, 0)$) following the energy price shock, is the source of the difference in pass-through, as it shifts the x further away from the optimal x^*, in regions where the adjustment probability given by $\Lambda(x)$ is higher.

7.5 Issues raised by this method

The main issue with this way of thinking of the pass-through, is that it tends to bias the estimation of the PT downward. Indeed, say that the steady-state adjustment rate is 2%, and the adjustment after a shock is 10%. It seems clear that the 8 ppts gap is attributable to the shock, and can therefore be seen as a full pass-through of these firms. Nevertheless, it seems very strange and makes no sense to think that the 2% of firms that would have adjusted anyway, are not passing 100% of the price increase, and therefore should indeed not be accounted for in the pass-through.
These firms would have adjusted anyway, and the next time they will do so will be completely independent from this shock, but the first time they adjust after the shock, they set a price such as to reach the new (post-shock) optimal price gap x^*. As a result, it would not be reasonable to take these adjusting firms out of the count, and estimate a pass-through of 8% instead of 10% for this particular period.
In general, it is possible to think that the first adjustment of a firm after a shock will correspond to a full pass-through of the MC increase at the firm level, and that once this shock is absorbed by this first price adjustment, the following adjustments will only be steady-state-like adjustments. Therefore, it appears that we need to take into account every post-shock increase in the price adjustment rates at impact, without neutralizing them through the removal of steady-state adjustment rates. On the other hand, in order to capture only the adjustments related to the shock, and thus an estimation of the pass-through, we need then to neutralize every further / later price adjustment. In a more qualitative sense, if we are interested in the speed of the pass-through, it may be sufficient to look at the first price adjustments, and see the rate at which the overall cumulative pass-through reaches 100%.

Remark : Note that there is no absolute guarantee that once the cumulative adjustment rate has reached 1, all firms have adjusted once. Indeed, it may be that some adjusted twice or more, and some did not, which would lead to an overestimation of the pass-through and its speed. Nevertheless we can think that this effect is not too large.

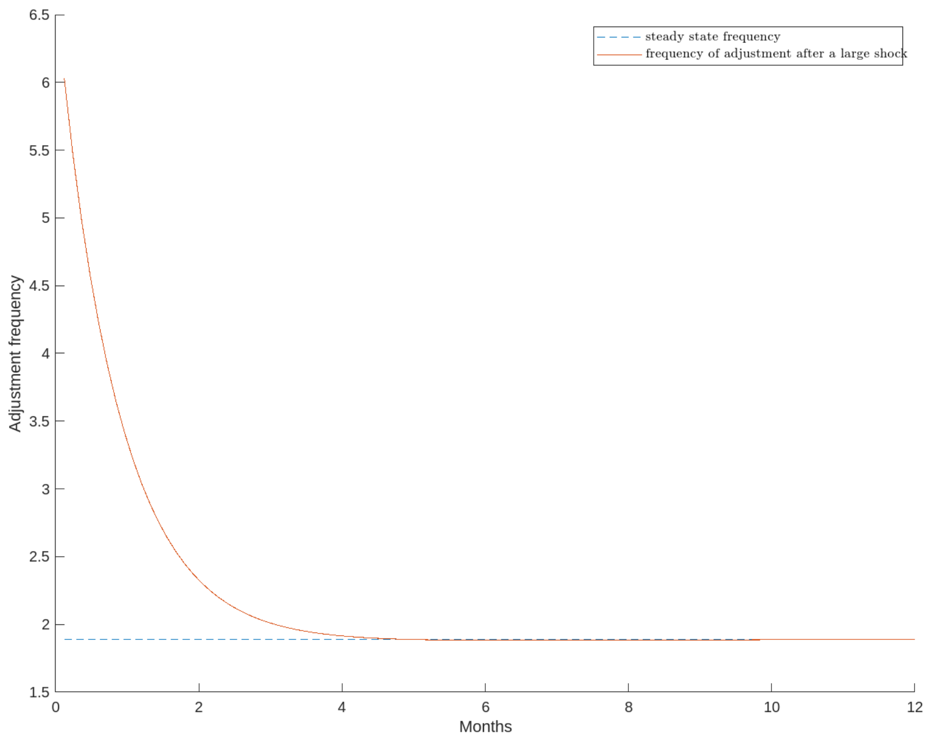

Figure 5: Period adjustment frequency