Modelling class attendance as a game of strategic interactions between students

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Abstract

Student attendance is a core concern to whomever addresses education policies. Both stakeholders and policy makers gain in a better understanding of its characteristics. Literature reveals correlations between attendance and grades, even though this link can be ambiguous. Many game theory models intended to represent teacher-student strategic interactions. This paper tackles the question in a novel way, as it approaches it from the perspective of students' decisions where courses are put in competition to self-studying at the school library. We build a simple baseline model close to a public good game, in which we introduce heterogeneity between students' utility of going to the library. We find that coming to class depends both on the workload and on student's individual reaction to workload, represented as types, and their expectations about other student's choices. Thereafter we extend it to allow for inter-temporal strategic interactions between students when the game is repeated and only some of the players have known type. We find that students whose type is unknown may be particularly encouraged to go to the library, as a way of signalling their type, and that even students who should be indifferent can be encouraged to go to the library due to their beliefs on the type of the other student. Finally we introduce a teacher whose utility depends on attendance and on a given objective workload that is considered optimal for students' learning. Her strategy regarding workload-setting and its impact on class attendance is evaluated.

1 Introduction

1.1 Motivation

The main idea behind this paper comes from the authors own student experiences. Throughout different courses of our curriculum, we witnessed a decrease in attendance in some during semesters. Classes would go from being full on first sessions to a low participation rate during exam periods and in the end of semesters. Our goal is to provide an explanation as to why this is the case. More specifically, our interrogations lie in the choices that students face when it comes to attending classes in opposition to self-studying. Some students argue that not going to class is a better use of their time for various reasons. More effective learning methods, less commuting time, going to the library to benefit from a good working environment or competition with urgent matters such as the workload for graded projects are examples of alternatives. This article presents our research and is structured as follows. First, we investigate existing literature engaging with student attendance in higher education institutions. Second, we draw the baseline model from which each extensions are then derived. They constitute part 3 and 4, and within them we investigate in detail the strategic interactions between agents, firstly among student then adding a teacher driven by her own utility.

1.2 Literature review

Massification of higher education after World War II caused major changes in the organization of universities and in the socioeconomic characteristics of students [7]. In the late seventies and eighties, emerged a field in economics which focused on understanding agents behaviours in academics. The main objective was to evaluate the means of education and find the most effective techniques to organize students curriculum. These goals are both targeted at individuals and on the aggregate level, especially since education has a direct impact on labor market and general economy. Thereupon, economists used the tools they knew and applied utility models to student and teachers [9, 10]. These studies unveiled the density of possibilities when it comes to modelling academic agents. One think of how the agents are represented, what is included in their preferences, how are learning outcomes measured, what are the goals of education policies etc. The richness in variety leaves room for evaluation of models with respect to real situation, especially when it comes to implementing policies. This diversity is both a chance and a concern. It is a chance because it gives space for a detailed analysis of the phenomenon and provides options for scientists for implementing their experiments. On the other hand, it is a source of possible complications and difficulty to establish clear conclusions. Moreover, one should be aware of the role that the design of models can play. It does not mean that results that are mainly driven by their design should be thrown out. We simply point out that considering the designs is a key element when analyzing these studies and that one should keep it in mind when drawing conclusion. Being mainly theoretical, our study falls in this category as we try to provide an explanation to what we observed throughout our students lives.

Focusing on teachers' side, studies try to understand what are professors motivations towards giving good classes. In 1975, Becker built a model where professors' time is allocated between research and teaching, depending on the output of both these activities, they thereafter consume[1]. Within this framework teachers have incentive to focus on the task out of which they can get the most and therefore concludes that improving teaching quality comes from increasing salaries associated with teaching. Many studies focused on what teaching methods should be implemented drawing a direct link with educational technology. A 2018 study explored a student-teacher game where teachers choose the rate of access to online course material and evaluates its impact on class attendance and grades. They show that full access incentives students to skip classes but has deterring effects on grades[5].

Considering students' problem, there are in facts many configurations to this problem. Indeed, students face many choices in a world where attending courses is not mandatory. Therefore one can represent them as utility maximizers. Courses are in competition with other activities such as paid work or leisure. This framework has the advantage of considering outside options and comprehending that students' lives are not driven only by their academical activities. Becker investigates such situations where students maximize their utility and find that increasing the frequency and quality reviewing of assessments has a positive effects on learning outcomes for students [2]. Looking at competing activities also allow for a better understanding of disparities between students regarding their socioeconomic situations. Poor students have more incentive not to attend class when compared to earning a salary as one compare marginal utilities.

When comparing utilities, one can also evaluate the outcome from going to class and hence concentrate how course quality affect attendance. It is indeed a reason produced by student surveyed by Romer as to why they would miss classes [11]. In a game theory model, this would imply the need to represent uncertainty over course quality through a variable encapsulating what students expect to receive from attending. This is not what we do in this paper as we focus on interactions between students and their choices regarding workload.

Attendance is not an obsession from higher education executives. Several evidences show its correlation with higher grades, grades being a proxy for student learning. In 1993, Romer investigated the impact of class attendance on grades and found that grades were positively correlated with attendance [11]. He gives hints as to why this might not be influenced by an omitted variable that would encapsulate motivation -meaning that motivated student would perform well and go to class, thus influencing the correlation between the variables. In 2013, Bratti and Staffolani looked at the effect of class attendance and self-study on grades and find that the two are highly correlated and mostly that attendance when controlled for self study does not explain grades [3]. Thus they provide a contradictory evidence to Romer's guess. Nonetheless was he right when affirming that comprehending the effect of student attendance was hard due to the intricate nature of considered variables. Such considerations have been put forward by Jaftha and Zahra Micallef [8] in their literature review, demonstrating that results from different studies regarding the effect of attendance on grades were to be nuanced. Such caution is required as many factors can influence the effect of attendance. In a 1983 study, Schmidt showed that the value of time spent studying was not homogeneous as some components were more effective than others, namely attending lecture, studying for midterm exam and attending discussion sessions[12].

Our paper originality lies in the fact that we address attendance from the students perspectives. Especially that we account for students' strategy and focus on how students spend their time dedicated to academical life. Schmidt's paper showed that student could allocate between various learning activities[12]. Therefore we investigate how they can maximize in this context. We also look for student-student interactions as we know that coming to class does not only depend on courses but also on social interactions[13]. Additionally we also try to incorporate a major point, in our student views, that is the varying nature of workload faced by students whether it is personal studying as exams arrive or mandatory at-home assignments given by professors. We found no trace of a similar design in the reviewed literature.

2 Baseline Game

2.1 Set up

The baseline game is meant to represent the trade-off for two students between attending a class or going to the library in order to spend time working on something else. Indeed, we consider that before the class, students are choosing whether to come to class or not, considering that they have a certain workload and could rather go to the library to fulfill their tasks and learn their lessons. We, thus, focus on an academic alternative to going to class, that is going to study at the library. The game is played by two representative students: Player 1 (P1) and Player 2 (P2). Both players decide whether to go to Class or to the Library and make their decision simultaneously. For now, we consider the strategies of each player for a given week of the semester.

We assume both players receive the same utility from attending the lecture which we normalize to 1. The intuition is that the content of the lecture always increases students' knowledge in the same way and we also state that players do not get disutility from being the only one attending, hence the constant payoff of 1. If a player goes to the Library, she gets some utility, this payoff is represented by the variable x. This payoff is also representative of the amount of work faced by student. We allow for this multiplicity of understandings as we assume a perfect correlation between the workload and the utility of self-studying. Thereafter we subtract 1 if they both go to the Library $(x-1)$, such that a player going to the library will prefer going alone rather than also having the other player going. For now, we assume $x \geq 0$ and is known by each player.

The idea behind these payoffs is that both players face the same workload and the utility of going to the library increases consequently. Hence, players would be better off studying in the Library when they have exams or a lot of homework. However, there is a cost associated to both players skipping class; this is interpreted as a punishment given by the professor to the class when no one is coming. Therefore, students know at the beginning of the game, that the teacher would punish them if both go to the library. Note that this negative payoff (-1) could also be interpreted as coming, at least partially, from players' feeling of guilt if they know that no one is coming to class. Overall, our baseline model can be seen as a member of the Public Good games family. It takes the following form:

	C_2	L_2
C_1	$1, 1$	$1, x$
L_1	$x, 1$	$x-1, x-1$

2.2 Simple Equilibria

We first look at the different equilibria of the game, for different values of x. First, it appears clearly that if $x \leq 1$, both players will always play C as a dominant strategy, and (C_1, C_2) will be a pure strategy Nash Equilibrium.

Proposition 1 If $x \leq 1$, (C_1, C_2) is a Nash Equilibrium. *Proof in A1.*

This result seems reasonable given our modelling assumptions. Indeed, students facing a low workload will have no incentive to study in the Library and everyone will attend lecture as its payoff is superior. This falls in line with what we observe during the first weeks of a semester when workload is low. In this case, C is a dominant strategy for all players.

Then for $1 < x < 2$, players do not have any strict dominant strategy and start mixing.

Proposition 2 If $1 < x < 2$, there are two pure strategy NE $\{(C_1, L_2), (L_1, C_2)\}$ and a mixed strategy equilibrium $\{((2-x)C_1 + (x-1)L_1, (2-x)C_2 + (x-1)L_2)\}$. *Proof in A2.*

Once again, these results seem reasonable given our modelling assumptions. We see that attendance starts decreasing when the workload accumulates. Therefore students begin to find working at the Library more attractive. In real situation, this is often the case after a few weeks throughout the semester.

Finally, if $x \geq 2$, both players will always play L as a dominant strategy, and (L_1, L_2) will be a pure strategy Nash Equilibrium.

Proposition 3 If $x \geq 2$, (L_1, L_2) is a Nash Equilibrium. *Proof in A3.*

This result show that there is a threshold after which students would never go to Class, since they have strong incentives to play L because of a high workload. In this case, L is a dominant strategy for both players. However, this is not what we observe in real life: as the semester goes by, and the workload increases, attendance decreases but there are still some students who prefer going to Class. Therefore, there would be an additive explanation at the individual level that would explain why some students go to Class despite a high value of x.

2.3 Introducing uncertainty on the students' types

From our observations, we see that some students always go to Class, other go sometimes and a few never go after the first lectures, as soon as the workload starts to increase. To complete our baseline model, let us represent these categories of students in our model by introducing types of students.

To do so, let us add a discount factor α_i on a student's utility of going to the library x, with α_i uniformly distributed on the interval $[0; 1]$. Types are randomly drawn and are private information; players know their α_i but ignore the one of the other.

	C_2	L_2
C_1	$1, 1$	$1, \alpha_2 x$
L_1	$\alpha_1 x, 1$	$\alpha_1 x - 1, \alpha_2 x - 1$

The intuition behind this discount factor α_i is that students react differently to the workload and have different payoffs for going to the library in consequence. There are several explanations that can support this assumption. The students may not need the same amount of time to achieve their work, some have low incentives to go to the library as they value less an extra hour of studying. This latter examples would be transcribed by a low α_i with effect that some students always choose C.

There remains uncertainty on the types of others. Thus players need to adapt their strategy to this situation of incomplete information. Clearly here, strategies depend monotonously on the type: The lower the α_i, the more likely the player is to play C. Thus, we look for an equilibrium in which players always play C if their type is lower than a fixed value of α_i (cut-off strategy) and L otherwise.

2.4 Equilibrium with types

Since we introduce incomplete information in our baseline model, we look for the optimal strategies in Bayesian Nash Equilibrium. In other words, we want the optimal strategy in a cut-off form. Note that for $x \leq 1$, playing C is a (weakly) dominant strategy; there is no cut-off. That is why we solve for $x > 1$. Remember $\alpha_i \sim U[0;1]$.

All strategies that are part of an equilibrium must be such that: if a player when she has type α_i^* plays C, then, if she has a α_i lower than α_i^*, her strategy should also be to play C.

So if $f_i(\alpha_i^*) = C$ then $f_i(\alpha_i) = C$ for all $\alpha_i < \alpha_i^*$.

We can now compute students' expected utility, at the cut-off:

$$\begin{aligned}\mathbb{E}[U_1(L_1)] &= \alpha_1^* x Pr(\alpha_2 < \alpha_2^*) + (\alpha_1^* x - 1) Pr(\alpha_2 > \alpha_2^*) \\ &= \alpha_1^* x \alpha_2^* + (\alpha_1^* x - 1)(1 - \alpha_2^*) \\ &= \alpha_1^* x - 1 + \alpha_2^*\end{aligned}$$

By symmetry : $\alpha_1^* = \alpha_2^* = \alpha^*$
In equilibrium: $\alpha_2^* = 2 - \alpha_1^* x$

Thus,

$$\begin{aligned}\alpha_1^* &= 2 - \alpha_2^* x \\ &= 2 - (2 - \alpha_1^* x)x \\ &= 2 - 2x + \alpha_1^* x^2 \\ &= \frac{2 - 2x}{1 - x^2}\end{aligned}$$

For $x > 1$, we obtain a cut-off form depending on the workload, which fits with our observations. We see that the threshold above which one chooses to go to the library decreases with x: as workload increases, the threshold value decreases for every agent. We can think of it the other way around, the higher the workload, the more players are likely to play L and skip classes (given α_i uniformly distributed).

Proposition 4 The Bayesian Nash Equilibrium is of the form:

$$f_i(c_i) = \begin{cases} \text{Class} & \text{if } \alpha_i \leq \alpha^* \\ \text{Library} & \text{if } \alpha_i > \alpha^* \end{cases}$$

In other words, if $\alpha_i > \alpha^*$, C is not a dominant strategy anymore; players anticipate that the other follows a similar strategy, given her type, and will play L.

3 First Extension: strategic interactions across periods between students

3.1 Motivation and set-up of the Extension

In this section, we introduce an extension to our model that aims to improve the accuracy of student decision-making dynamics. The original model depicted student interactions in a single time frame, considering beliefs about peers' abilities and workload changes over the semester. This extension adds inter-temporal learning among students, transforming the model into a signaling game. Over successive class sessions, students observe and adapt to each other's actions, refining their strategies.

Rather than analyzing the entire semester, which could be cumbersome, we focus on periods with a significant workload that allows for mixing strategies and for different strategies across student types. Indeed, as denoted in the baseline, when the workload is too low, there is no room for mixing and uncertainty about the other student's strategy. To keep the model manageable and interpretable, we limit the number of iterations. We find that two repetitions adequately capture the dynamic learning processes in signaling games.

Additionally, this extension diverges from the baseline by introducing incomplete information only about the type of Student 1, who can be either of low type ($\alpha_L = 0$) or high type ($\alpha_H = 0.5$). One of the reasons for choosing 0.5 as the value for a type high student is that it represents a median student, who can be seen as a representative agent. On the other hand, being of type low ($\alpha_L = 0$) would indicate that you are part of those students who always come to class, and thus never respond to workload increases. The introduction of this particular type of student is motivated by empirical observations, done throughout the semesters, that some students always come to class, whatever the workload. Student 2's type is known, while Student 1's type is only known to herself. Finally, we choose a workload of $x = 3$ in period 1 and $x = 3.5$ in period 2, that corresponds to intermediary weeks of the semester, as motivated before, to allow for a maximum uncertainty and strategic interactions.

3.2 Normal Form of the Game

The previous explanation leads to the following normal form Game:

If P_1 is type α_L:

- Period A: $x = 3$

	C_2	L_2
C_1	$1, 1$	$1, \frac{3}{2}$
L_1	$0, 1$	$-1, \frac{1}{2}$

- Period B: $x = 3.5$

	C_2	L_2
C_1	$1, 1$	$1, \frac{7}{4}$
L_1	$0, 1$	$-1, \frac{3}{4}$

If P_1 is type α_H:

- Period A

	C_2	L_2
C_1	$1, 1$	$1, \frac{3}{2}$
L_1	$\frac{3}{2}, 1$	$\frac{1}{2}, \frac{1}{2}$

- Period B

	C_2	L_2
C_1	$1, 1$	$1, \frac{7}{4}$
L_1	$\frac{7}{4}, 1$	$\frac{3}{4}, \frac{3}{4}$

Before solving the game and looking at the equilibria, we need to define some parameters. Let,

- β be the prior belief of player 2 on player 1 type being a type low student.
- q_2 be the probability that P2 plays C_2
- p_2 be the probability that P1 plays C_1

The probability are different across periods, types and beliefs, therefore we will specify each time the particular configuration.

3.3 Perfect Bayesian Equilibria

In this section, we will state the equilibrium and then prove it by checking for any profitable deviations. Some steps are not detailed here *(details in A4)*.

Proposition 5 If $\beta < \frac{1}{2}$, then there is the following separating equilibrium:

$$\text{In Period A}\begin{cases}\text{P2 plays } C_2\\ \text{P1 plays } C_1 \text{if she is } \alpha_L\\ \text{P1 plays } L_1 \text{if she is } \alpha_H\end{cases}$$

$$\text{In Period B}\begin{cases}\text{P2 plays } L_2 \text{ if in A P1 played } C_1\\ \text{P2 plays } (\frac{1}{4}C_2, \frac{3}{4}L_2)\text{if in A P1 played } L_1\\ \text{P1 plays } \ C_2 \text{ if she is } \alpha_L\\ \text{P1 plays } (\frac{1}{4}C_2, \frac{3}{4}L_2) \text{ if she is } \alpha_H\end{cases}$$

Proof To check this is an equilibrium, let us examine if any player can improve their payoff by deviating from these strategies. Notice, that in this case, we are facing a separating equilibrium.

- Period B
  - Player 1's Deviation:

  If P1 is α_L she will never deviate since C_1 is a dominant strategy. And if she is α_H playing $(\frac{1}{4}C_1, \frac{3}{4}L_1)$ will gave him a pay-off of 1, and by definition of the indifference condition, there are no profitable deviation.

  - Player 2's Deviation:

  If P1 played C_1 in the previous period, P2 knows that she is of type α_L and that C_1 is a dominant strategy. Thus playing L_2 maximize her pay-offs. And if P1 played L_1 in the previous period, P2 knows that she is of type α_H. And playing $(\frac{1}{4}C_1, \frac{3}{4}L_1)$ will gave him a pay-off of 1, and by definition of the indifference condition, there are no profitable deviation.

- Period A
  - Player 1's Deviation:

  If P1 is α_L she will never deviate since C_1 is always a dominant strategy.

  If she is α_H playing L_1 maximizes her pay-offs since player 2 is going to class. Furthermore, if player 2 observes Player 1 choosing C_1, she would go to the library in the following period for sure. So there is no profitable deviation for Player 1.

  - Player 2's Deviation:

Let us compute - and compare - Player 2's expected utility in case of deviation in Period 1:

$$\begin{aligned}\mathbb{E}U(L_1) &= \frac{3}{2}P(\text{P1 goes to class}) + \frac{1}{2}P(\text{P1 goes to the library}) \\ &= \frac{3}{2}P(\alpha_1 = \alpha_L) + \frac{1}{2}P(\alpha_1 = \alpha_H) \\ &= \frac{3}{2}\beta + \frac{1}{2}(1-\beta) \\ &= \beta + \frac{1}{2}\end{aligned}$$

On the other hand, $\mathbb{E}U(C_1) = 1$. Therefore, $\mathbb{E}U(C_1) > \mathbb{E}U(L_1) \iff \beta < \frac{1}{2}$. In this case there is no profitable deviation for Player 2.

As there are no profitable deviations for either player, the proposed strategy profile constitutes a perfect Bayesian equilibrium when $\beta < \frac{1}{2}$.

Proposition 6 If $\beta \geq \frac{1}{2}$, then there is a pooling-type equilibrium:

$$\text{In Period A} \begin{cases} \text{P1 plays } C_1 \\ \text{P2 plays } L_2 \end{cases}$$

$$\text{In Period B} \begin{cases} \text{P1 plays } C_1 \\ \text{P2 plays } L_2 \end{cases}$$

Proof Now, we need to examine if any player can improve their payoff by deviating from these strategies. Notice, that in this case, we are facing a pooling equilibria.

- Period B
  - Player 1's Deviation:

  If P1 is α_L she will never deviate since C_1 is a dominant strategy. And if she is α_H playing $(\frac{1}{4}C_1, \frac{3}{4}L_1)$ is not an optimal strategy anymore since P2 will always play L_2 and she will always play C_2. Thus, there are no profitable deviation.

  - Player 2's Deviation:

  Since here, we are in a pooling equilibrium, whether P1 played C_1 or L_1 in the previous period, P2 doesn't know the type of P1 and can't update her beliefs about β. Therefore, by indifference condition computation, P2 plays C_2 if $\frac{1-4\beta}{4(1-\beta)} \geq p_2^H \Leftrightarrow$ Plays C_2 if $\beta \geq \frac{1}{4}$(see, appendix A4 for the detailed computation). Thus, here, playing L_2 maximize P2 pay-offs. There are no profitable deviations.

- Period A
  - Player 1's Deviation:

  If P1 is α_L she will never deviate since C_1 is always a dominant strategy.

  If she is α_H playing C_1 maximizes her pay-offs since player 2 is not going to class. Furthermore, in any case, P2 is playing L_2 in period 2, thus deviating will not increase payoffs in period 2. So there is no profitable deviation for Player 1.

  - Player 2's Deviation:

  By indifference condition, P2 plays C_2 if $\frac{1-2\beta}{2(1-\beta)} \geq p_2^H \Leftrightarrow$ Plays C_2 if $\beta \geq \frac{1}{4}$(see appendix A4 for the details). Thus, here, playing L_2 maximize P2 pay-offs.

  Let us compute - and compare - Player 2's expected utility in case of deviation in Period 1:

$$
\begin{aligned}
\mathbb{E}U(L_1) &= \frac{3}{2}P(\text{P1 goes to class}) + \frac{1}{2}P(\text{P1 goes to the library}) \\
&= \frac{3}{2}P(\alpha_1 = \alpha_L) + \frac{1}{2}P(\alpha_1 = \alpha_H) \\
&= \frac{3}{2}\beta + \frac{1}{2}(1-\beta) \\
&= \beta + \frac{1}{2}
\end{aligned}
$$

  On the other hand, $\mathbb{E}U(C_1) = 1$. Therefore, $\mathbb{E}U(L_1) > \mathbb{E}U(C_1) \iff \beta > \frac{1}{2}$. In this case there is no profitable deviation for Player 2.

As a result, it appears that in equilibrium, a student whose type is unknown will have an incentive to reveal her type perfectly under certain condition ($\beta < \frac{1}{2}$). Otherwise, she would prefer to be in a pooling-type equilibrium, where the other player cannot learn anything on her type. Most interestingly, this allows us to see that, in both cases, there is on average at least one student going to the library. This is the case even though with the types that were chosen, some students should be mixing in the first period and the others going to class. Furthermore, a student whose type is only privately known, may have - in a separating equilibrium - a strong incentive to go to the library in the first period, even when she is of high type and would rather mix in a one period model (as she is at the cut-off).

Therefore, studying these interactions among different periods shows that students generally have strong incentive to skip classes and go to the library, these incentives being sometimes even increased by students' expectations about the way other students may behave. Introducing this sequentiality allows us to emphasize the importance of interactions in this game. Indeed, students' decisions really do not only depend on their type and the workload that is imposed, but also on their interactions and beliefs about other players' strategies. There appears to be no way to increase attendance only by looking at students' interactions.

4 Second Extension: Introducing a Teacher

4.1 General form of the game with Teacher

Introducing a teacher in our model allows for a detailed study of new equilibria. We implement it in order to investigate the possibility of different outcomes compared to previous forms of the game. Correa and Gruver showed that many game in academia, being in the set up of public good, produce under effective equilibria [4]. We therefore look at how the teacher can have an impact to improve the situation. Indeed, until now the workload, which plays an important role in our model, has been considered as being purely exogenous. Nevertheless, in reality it is set by a teacher who is choosing the level of x to impose on students. The teacher can be seen as a particular kind of what macroeconomists often refer to as "a benevolent central planner". Indeed, the teacher will set a particular workload, that she sees as optimal regarding student's learning process, which in turn will impact the outcome of the game, that is attendance. Knowing the game in which the students are then playing, the teacher is able to anticipate their responses to any given workload she may give.

As we have seen in the baseline game and the first extension, attendance is usually uncomplete and students have incentives to go to the library instead of coming to class. Furthermore, there appeared to be no "good way" to implement student-level strategy to remedy this issue, and the incorporation of sequentiality led to signalling equilibria where players were sometimes even less incentivized to attend class. Introducing a teacher in the model could therefore be a way to have another look at these issues, and try to increase class attendance using the tools she has at hands.

In this extension, we will consider that the teacher has knowledge over the optimal workload that should be set, in order to make the students learn and get the best from her class and teaching. Indeed, thanks to her experience and knowledge about the way students learn in her class, she knows how to set an optimal workload. This assumption appears relatively plausible and can also be a remedy to some disparities among students [6, 5]. In addition, she values the overall class attendance. In order to keep it simple, we will keep this example with 2 students, even though the mechanism could be extended to a class of 20 students, as it does not affect greatly the outcome, the underlying mechanisms and the intuition. Clearly, as we have seen that greater workload has negative effect on class attendance, a trade-off emerges for the teacher between those two elements.

4.2 Teacher problem

Let us think of the problem in the following way. In general, the teacher is trying to maximize her utility $U(x, N)$, which is a function of the workload that is set and the attendance in class. This function takes the following (additive) form:

$$U(x, N) = v(x) + \phi w(N)$$

Utility is here obtained from the workload (x) and class attendance (N), in two different ways described by the functions v and w. Additionally, ϕ represents the trade-off that is to be made between class attendance and workload, and will be useful later on. Further research and literature could focus on the way to describe precisely these two functions and the way teachers attach utility to class attendance and workload choice. In this case, for the sake of intuition (and simplicity) let us consider a simple, linear, form of this utility function.

Let us consider that workload only takes integer values (indeed the teacher can give 2 or 3 exercises to solve, but not 2.576 for instance), and the teacher finds it optimal to put a workload of $x^* = 5$. It is trivial that x cannot exceed this value (as it would decrease the teacher's utility from x and from class attendance). Therefore x has support $[\![0, 5]\!]$. As there are two students, the N can only take values 0; 1; 2.

$$U(x, N) = x + \phi N$$

In this simple form utility comes from workload and attendance in a linear fashion. The higher the workload, the closer to the optimal level $x^* = 5$, the higher the utility. Furthermore, there is an additional term ϕ, which traduces how much the teacher values class attendance relative to workload. The higher ϕ, the more the teacher values students' attendance relative to setting a workload close to the optimal. Therefore, it can theoretically take any positive value, even though we will see that it is only necessary to study it for relatively low values (between 0 and 8 or 10).

4.3 Solving the Teacher's problem

We can now solve for the teacher's strategies, knowing that students react strategically in the way described in the baseline. To do so, let us compute the expected utility she can get from the different workload that can be set:

$$\mathbb{E}U(x,N) = \mathbb{E}(x+\phi N) = x + \phi\mathbb{E}(N) = x + \phi[2P(N=2) + P(N=1) + 0]$$

First, note that when $x = 0$ and $x = 1$, as we saw in the baseline, coming to class is a (at least weakly) dominant strategy for all types of students. Therefore, in these cases, $N = 2$. Then, let us look for the teacher's expected utility in case of higher workload.

The students follow the strategy that was described in the baseline, therefore we know that one comes to class if her or her type $\alpha < \alpha^*$. The teacher, playing first, knows and can anticipate these responses. As she can only choose the workload, her utility is only a function of x, which will also show up in the students' cutoffs. By symmetry, all students have the same cut-off $\alpha^* = \frac{2-2x}{1-x^2}$. As a result (and by property of a binomial distribution), the teacher's expected utility becomes :

$$\mathbb{E}U(x) = x + \phi[2(\alpha^*)^2 + 2\alpha^*(1-\alpha^*)] = x + 2\phi\alpha^*$$

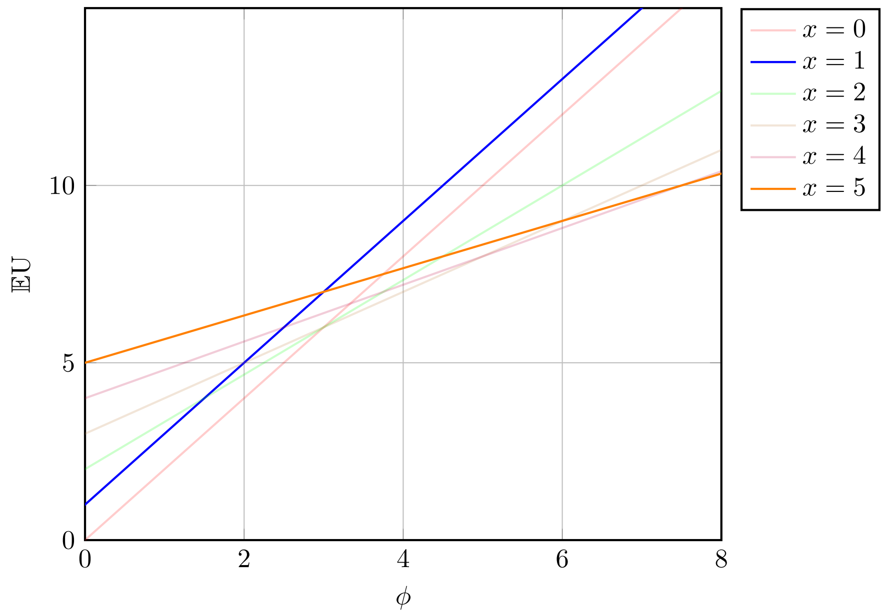
We can now compute the teacher's expected utility for the different values that x can take (between 0 and 5). The only unknown still appearing is now the ϕ coefficient of the teacher.

Table 1: Teacher's expected utility, for different workloads

$Workload(x)$	$\mathbb{E}U(x)$
0	2ϕ
1	$2\phi + 1$
2	$\frac{4}{3}\phi + 2$
3	$\phi + 3$
4	$\frac{4}{5}\phi + 4$
5	$\frac{2}{3}\phi + 5$

In table 1 we compute the expected utility associated to different workloads, as a function of ϕ. In order to find the best response of the teacher, we now have to compare these and see what is the best response of a teacher maximizing her utility, depending on the value of ϕ.

Figure 1: Teacher's expected utility for different workloads, as a function of ϕ



4.4 Equilibria

In figure 1, the behavior of the teacher's expected utility, for different workloads, on the support of ϕ are plotted. From this, it is clear that only two possible strategies can be dominant for the teacher (see A5).

$$f(\phi) = \begin{cases} x = 5 & \text{if } \phi < 3 \\ x = 1 & \text{if } \phi > 3 \\ \text{Indifferent} & \text{if } \phi = 3 \end{cases}$$

Proposition 7 The best strategy of the teacher only depends on ϕ, how much she values attendance relative to setting a workload close to her optimal one.

Proposition 8 Above a certain value of ϕ, her dominant strategy is no longer to set $x = 5$ but $x = 1$. In this situation, the expected number of students N coming to class is 2, and is way greater than the $\mathbb{E}(N) = 2/3$ associated with a workload $x = 5$.

Note that, from table 1, we can even see that if the optimal workload of the teacher was below 5, setting x equal to 1 is a pure dominant strategy, no matter the value of ϕ.

The form of the utility functions could be described differently and refined in further research, but the main conclusions are here: If the teacher values more attendance relative to workload above a certain threshold (here 3 times more), she should set a workload that is lower than her optimal one, therefore leading attendance to increase a lot compared to other equilibria. In fact, she would be setting the higher workload that allows her to have full class attendance ($N = 2$), that is $x = 1$. This way, we have shown that as soon as a teacher is willing enough to increase attendance, at the expense of the high workload she would like to set, it is possible to reach an equilibrium in which all students are coming to class, whatever their type.

It is important to mention that this is not the only way to increase attendance, and that the teacher could use different tools as previously mentioned [4, 5]. One can also increase the punishment on non-attending student with sanctions regarding their academic journey. It would mechanically lead to higher cut-off α^* and therefore higher average/expected attendance for any given workload level. She could also implement tools to make the deviation (to library) more costly, not by increasing the punishment, but by increasing the incentive to go to class. There we fall back on the the part of literature introducing uncertainty on quality of the class that was discussed in the literature review. Or even by announcing that if there is only one student in class, exclusive course material will be given, providing attending students with an advantage.

5 Conclusion

Following an established trend in literature, we used utility maximization models and game theory to represent real life situations. Even though links have been established between attendance and grades, more recent investigations confirm the intertwined nature behind the understanding of attendance and questioned the rigorousness of its links with academical success. These unsatisfactory conclusion drove us toward exploring new reasons to explaining attendance. Our contribution to this field lies in the novelty of our approach in the way we represent students and their interactions. Doing so we provide a new way to consider real situations and participate in the disentanglement of the exposed knot.

Our model provides insights into the relationship between workload, student behavior, and strategic interactions in educational environments. We observe a distinct threshold, dependent on workload, influencing students' decisions to attend classes or opt for self-study at the library. We notice that, as workload intensifies, the threshold for choosing library study diminishes across all student types; making students more prone to skip classes during more demanding academic periods.

Moreover, we uncover the strategic implications of students' knowledge about their peers' preferences. Indeed, under certain conditions, students with undisclosed types are incentivized to reveal their preferences, leading to equilibria where at least one student consistently opts for library study. Sequential interactions emphasizes even more on the importance of strategic thinking in student decision-making. We explain why students' choices are not solely dictated by individual preferences and workload but are also influenced by beliefs about peer behavior.

Thus, our results suggest that efforts to increase class attendance solely through student interactions may not be effective. Instead, the optimal strategy for the teachers would depend on their valuation of attendance relative to workload. Our proposition demonstrates that, above a certain threshold value, setting a lower workload emerges as the dominant strategy for maximizing attendance, and teacher's utility. This insight challenges conventional approaches to managing class attendance and underscores the need for a nuanced understanding of teacher-student dynamics in order to maximize student engagement and learning outcomes.

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A Appendix

A.1

Proof We check for deviations:

Denote p the probability of P1 playing C_1 and q the probability of P2 playing C_2.

- P1
  - No deviation: $\mathbb{E}[U_1(C_1)] = 1$
  - Deviation, i.e playing L: $\mathbb{E}[U_1(L_1)] = qx + (1-q)(x-1) = x + q - 1$
    Then: $1 > x + q - 1$
- P2

  Symmetric payoffs: $\mathbb{E}[U_2(C_2)] > \mathbb{E}[U_2(L_2)]$

A.2

Proof We check for deviations from (C_1, L_2):

- P1
  - No deviation: $\mathbb{E}[U_1(C_1)|L_2] = 1$
  - Deviation, i.e playing L: $\mathbb{E}[U_1(L_1)|L_2] = x - 1$
  - Then: $1 > x - 1$
- P2
  - No deviation: $\mathbb{E}[U_2(L_2)|C_1] = x$
  - Deviation, i.e playing L: $\mathbb{E}[U_2(L_2)|C_1] = 1$
  - Then: $x > 1$

Payoffs are symmetric, so there is no profitable deviation from (L_1, C_2).

We use the indifference condition to get the mixed strategy equilibrium:

- P1
  - $\mathbb{E}[U_1(C_1)] = 1$
  - $\mathbb{E}[U_1(L_1)] = qx + (1-q)(x-1)$
  - Then: $q = 2 - x$
- P2
  - By symmetry: $p = 2 - x$

Thus, in the mixed strategy equilibrium, both players randomize their strategy by choosing C with probability (2-x) and L with probability (x-1).

A.3

Proof We check for deviations:

- P1
  - No deviation: $\mathbb{E}[U_1(L_1)] = qx + (1-q)(x-1) = x - 1 - q$
  - Deviation, i.e playing C: $\mathbb{E}[U_1(C_1)] = 1$ Then: $x - 1 - q > 1$
- P2
  - Symmetric payoffs: $\mathbb{E}[U_2(C_2)] > \mathbb{E}[U_2(L_2)]$

A.4

Proposition 5 If P1 is α_L, then she would play C_1 in both periods.

Proof It appears directly that, in period B, P1 would play C_1 since it is a dominant strategy.

$$\mathbb{E}[u_1(C_1, \sigma_2^{\text{NE}})] > \mathbb{E}[u_1(L_1, \sigma_2^{\text{NE}})]$$

Given that in period B she consistently chooses strategy C_1, player P1 will also select strategy C_1 in period A because it is a dominant strategy, ensuring there are no profitable deviations available.

$$\mathbb{E}[u_1(C_1, \sigma_2^{\text{NE}})] > \mathbb{E}[u_1(L_1, \sigma_2^{\text{NE}})]$$

Proposition 6 : If P1 played L_1 in period A, then in Period B the sub game Bayesian Nash Equilibrium's are (C_1;L_2), (L_1;C_2), ($\frac{1}{4}C_1$, $\frac{3}{4}L_1$; $\frac{1}{4}C_2$, $\frac{3}{4}L_2$) .

Proof Since P1 always plays C_1 when she is of type low, choosing L_1 would reveal to P2 that she is of type α_H, in the case of a separating equilibrium. Consequently, in Period B P2 would completely update her beliefs and β would be equal to 0. Without the original uncertainty, it appears clearly that (C_1;L_2), (L_1;C_2) are sub-game NE. Now, looking for mixed strategies:

$$\begin{aligned} u_1(C_1, \sigma_2^{\text{NE}}) &= u_1(L_1, \sigma_2^{\text{NE}}) \quad \textit{(Indifference condition)} \\ \Leftrightarrow \frac{1}{4} &= q_2^H \\ \frac{1}{4} &= p_2^H \quad \textit{(symmetrically)} \end{aligned}$$

Proposition 3 : If we are in pooling equilibrium, P2 can't update her believes in period B and P2 plays C_2 if $\frac{1-4\beta}{4(1-\beta)} \geq p_2^H$.

Proof

$$
\begin{aligned}
\mathbb{E}u_2(\sigma_2^{\text{NE}}, C_1) = \mathbb{E}u_2(\sigma_2^{\text{NE}}, L_1) \quad \textit{(Indifference condition)} \\
\Leftrightarrow \beta + (1-\beta)(p_2^H + 1 - p_2^H) = \beta\frac{7}{4} + (1-\beta)(\frac{7}{4}p_2^H + (1-p_2^H)\frac{3}{4}) \\
\Leftrightarrow \frac{1-4\beta}{4(1-\beta)} = p_2^H
\end{aligned}
$$

A.5

Let us show when teacher's utility from setting $x = 1$ exceed the one she gets from setting $x = 5$, that is for which values of $\phi : U(1) > U(5)$.

$$
U(1) > U(5) \Leftrightarrow 1 + 2\phi > 5 + \frac{2}{3}\phi \Leftrightarrow \phi > 3
$$